VERSITY OF ILLINOIS
ULLETIN

Vol. 44 March 23, 1947 No. 45

ENGINEERING EXPERIMENT STATION
BULLETIN SERIES No. 367

INFLUENCE CHARTS FOR COMPUTATION OF VERTICAL DISPLACEMENTS IN ELASTIC FOUNDATIONS

BY

NATHAN M. NEWMARK

PRICE: FORTY CENTS

PUBLISHED BY THE UNIVERSITY OF ILLINOIS
URBANA

Published every five days by the University of Illinois. Entered as second-class matter at the post office at Urbana, Illinois, under the Act of August 24, 1912. Office of Publication, 358 Administration Building, Urbana, Illinois. Acceptance for mailing at the special rate of postage provided for in section 1103, Act of October 3, 1917, authorized July 31, 1918.

THE Engineering Experiment Station was established by act of th ... on De-
cem ... onduct
investigatio ... eering,
manufactur ... of the
State.

The mar ... vested
in an Execu ... it, the
Heads of th ... g, and
the Profess ... le for
the establish ... ation,
including th ... ers of
the teaching ... entific
research, ei ... Corps,
composed of ... stants,
and special ...

To rend ... ble to
the public, ... d dis-
tributes a s ... ars of
timely inter ... from
various so ... ntele
of the Stat ... press
written by ...

The vo ... e are
merely arb ... f the
University ... f the
Engineerin ... which
should be ...

For cop ...

... IS,

... INOIS

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION
BULLETIN SERIES No. 367

INFLUENCE CHARTS FOR COMPUTATION OF VERTICAL DISPLACEMENTS IN ELASTIC FOUNDATIONS

BY

NATHAN M. NEWMARK
Research Professor of Civil Engineering

PUBLISHED BY THE UNIVERSITY OF ILLINOIS

PRICE: FORTY CENTS

3000—3-47—34772

UNIVERSITY
OF ILLINOIS
:: PRESS ::

ABSTRACT

This bulletin describes a simple graphical procedure for computing vertical displacements at the surface or within the interior of an elastic, homogeneous, isotropic solid body bounded by a plane horizontal surface and loaded by distributed vertical loads at the surface. The displacements are computed from charts given herein merely by counting on the chart the number of elements of area, or blocks, covered by a plan of the loaded area drawn to proper scale and laid upon the chart.

CONTENTS

LIST OF PLATES

LIST OF TABLES

INFLUENCE CHARTS FOR COMPUTATION OF VERTICAL DISPLACEMENTS IN ELASTIC FOUNDATIONS

I. Introduction

1. *Scope of Bulletin.*—This bulletin, a sequel to Bulletin 338,[1] describes a simple graphical procedure for computing vertical displacements at the surface or within the interior of an elastic, homogeneous, isotropic solid body bounded by a plane horizontal surface and loaded by distributed vertical loads at the surface. The displacements are computed from charts given herein merely by counting on the chart the number of elements of area, or blocks, covered by a plan of the loaded area drawn to proper scale and laid upon the chart. The charts are used in essentially the same manner as those for computation of stress given in Bulletin 338.

Three influence charts are included. Plate 1, for computing the vertical displacement or settlement at the surface, can be used for any value of Poisson's ratio. Plate 2, for computing the vertical displacement at any depth Z beneath the surface, is used directly for a value of Poisson's ratio of 0.5. Plate 3 is a chart for determining the correction to be used with the results from Plate 2 for values of Poisson's ratio other than 0.5. The influence values for the individual squares in each chart are dependent on the value of Poisson's ratio, on the intensity of loading, on the base length L or the depth Z, and on the modulus of elasticity.

Table 1 contains numerical data for constructing the charts. Chapter III gives formulas for computing the numerical values.

2. *Acknowledgments.*—The investigation reported herein was conducted as a part of the work of the Engineering Experiment Station of the University of Illinois, of which Dean M. L. Enger is the director, and of the Department of Civil Engineering, of which Professor W. C. Huntington is the head.

The selection of the numerical values for drawing the charts in Plates 1, 2, and 3 was governed by considerations involved in the use of the charts by the United States Waterways Experiment Station, Vicksburg, Mississippi, for computing vertical displacements in the subgrade of airfield pavements. Acknowledgment is made to the Director of the Station for permission to publish these charts.

---

[1] N. M. Newmark, "Influence Charts for Computation of Stresses in Elastic Foundations," Univ. of Ill. Eng. Exp. Sta. Bul. 338, 1942.

3. *Notation.*—The following notation is used:

p = intensity of load, or load per unit of area.
s_o = settlement or vertical deformation of a point at the surface.
s' = settlement of a point at the depth Z beneath the surface.
E = modulus of elasticity, or modulus of deformation of the material.
L = base length for settlement at surface.
Z = depth of stratum at which settlement is computed.
μ = Poisson's ratio.
n_o = number of influence squares (blocks) from Plate 1.
n' = number of influence squares (blocks) from Plate 2.
n_c = number of influence squares (blocks) from Plate 3.
N = sum of the number of blocks within a circle of radius r.
r = radius of loaded circular area.
α = arc tan r/Z.

II. Use of Influence Charts

4. *Outline of Procedure for Use of Charts.*—The influence charts given herein are used in the same manner as those described in Bulletin 338 for the computation of stress.

To use the charts, a drawing is made of the loaded area to such a scale that the depth Z at which the settlement is desired (in Plates 2 and 3) or the base length L (in Plate 1) is, to scale, equal to the length denoted by Z or by L on the plate. The drawing of the loaded area is then placed on the chart in such a way that the origin of the chart falls under the point at which the settlement is desired.

The number of blocks covered by the loaded area is then counted. The settlements are determined from Equations (1) and (2) below. The calculations can be made directly only for uniformly loaded areas. Where the area is not uniformly loaded the charts can still be used by considering the non-uniform loading to be made up of several sets of uniformly loaded areas.

The equations for determining the settlement are as follows:

$$s_o = 0.02\,(1 - \mu^2)\, n_o\, pL/E \tag{1}$$

$$s' = 0.01\,(1 + \mu)\,[n' + (1 - 2\mu)\, n_c]\, pZ/E. \tag{2}$$

For the special case where $\mu = 0.5$ these equations become

$$s_o = 0.015\, n_o\, pL/E \tag{3}$$

$$s' = 0.015\, n'\, pZ/E. \tag{4}$$

For a given loaded area, drawings of the plan of the loaded area to different scales are required in order to compute settlements at various depths. For settlement at the surface, however, the loaded area may be drawn to any scale whatsoever and the length L determined for the particular scale used.

All the charts are radially symmetrical; therefore the loading plan may be rotated through any angle about a vertical axis through the origin of the chart without changing the magnitude of the settlement.

In using the charts, parts of blocks may be estimated with sufficient accuracy for practical purposes. In general the loaded area will be drawn on tracing paper and laid upon the chart so that blocks may be counted through the tracing.

The charts may be used to compute the change in thickness of a stratum, by determining the vertical displacement of the top and bottom boundaries of the stratum.

5. *Illustrative Example.*—As an example of the use of the charts, consider the problem of a loaded area 100 ft. square, with $E =$ 1,000,000 lb. per sq. ft., $\mu = 0.5$, and $p = 2{,}000$ lb. per sq. ft. It is required to find the settlement at the corner of the area, and also the settlement vertically beneath the corner of a stratum at a depth of 25 ft.

Since Plate 1 can be used with any scale, it is convenient to use the same drawing of the loaded area for both Plate 1 and Plate 2. Consequently the plan of the loaded area is drawn to such a size that the length of a side is four times the length Z in Plate 2 or four times the length L in Plate 1.

The following results are obtained:

$$n_o = 112,$$

$$s_o = (0.02)\,(0.75)\,\frac{(2000)\,(25)}{1{,}000{,}000}\,(112)\text{ ft.} = 0.084\text{ ft.};$$

$$n' = 109, \qquad n_c = 0,$$

$$s' = (0.01)\,(1.5)\,\frac{(2000)\,(25)}{1{,}000{,}000}\,(109)\text{ ft.} = 0.082\text{ ft.}$$

III. Formulas and Numerical Values for Constructing Charts

Consider a circular area of radius r loaded uniformly by a distributed load of intensity p. The vertical displacement s of a point at a distance Z beneath the center of the circular area is given by the following formula:

$$s = (1 + \mu)\frac{pr}{E}\left\{\sin\alpha + (1 - 2\mu)\frac{1 - \cos\alpha}{\sin\alpha}\right\} \tag{5}$$

where

$$\tan\alpha = \frac{r}{Z}. \tag{6}$$

For the surface of the loaded area $Z = 0$ and, consequently, $\alpha = \pi/2$. Equation (5) then becomes

$$s = 2(1 - \mu^2)\frac{pr}{E} = 2(1 - \mu^2)\left(\frac{pL}{E}\right)\left(\frac{r}{L}\right). \tag{7}$$

L is introduced in Equation (7) merely for convenience in having a base length or scale for the plate.

The values of r/L in Equation (7) or of $r/Z = \tan\alpha$ in Equation (5) are tabulated in Table 1 for various values of N, the number of blocks within a circle of radius r. In other words, the increment in N between any two values of r gives the number of blocks n_o, n', or n_c between two circles having as radii the two values of r.

For deflections at the surface, the value of N is obtained by comparing Equations (1) and (7) and is given by the equation

$$N = 100\frac{r}{L}. \tag{8}$$

For deflections beneath the surface the value of N corresponding to n' or n_c is obtained by comparing Equations (5) and (2). For n' it is:

$$N = 100\frac{\sin^2\alpha}{\cos\alpha}. \tag{9}$$

Table 1

Values of $\frac{r}{L}$ and $\frac{r}{Z}$ for Vertical Deflection

N	Increment in N	Number of Radial Segments in ¼ Circle	$\frac{r}{L}$ for n_o, Deflection at Surface	$\frac{r}{Z}$ for n', Deflection at Depth Z, $\mu = \frac{1}{2}$	$\frac{r}{Z}$ for n_c, Deflection at Depth Z, Correction for μ
0	0		0	0	0
4	4	1	0.04	0.202	0.286
12	8	2	0.12	0.357	0.504
20	8	2	0.20	0.470	0.663
40	20	5	0.40	0.698	0.980
60	20	5	0.60	0.898	1.249
80	20	5	0.80	1.089	1.497
100	20	5	1.00	1.272	1.732
140	40	10	1.4	1.640	2.182
180	40	10	1.8	2.010	2.615
220	40	10	2.2	2.385	3.04
260	40	10	2.6	2.765	3.46
300	40	10	3.0	3.15	3.87
340	40	10	3.4	3.53	4.28
380	40	10	3.8	3.92	4.69
420	40	10	4.2	4.31	5.10
460	40	10	4.6	4.70	5.51
500	40	10	5.0	5.10	5.92
600	100	25	6	6.08	6.93
700	100	25	7	7.07	7.94
800	100	25	8	8.06	8.94
900	100	25	9	9.05	9.95
1000	100	25	10	10.05	10.95
1100	100	25	11	11.04	11.96
1200	100	25	12	12.04	12.96
1300	100	25	13	13.04	13.96
1400	100	25	14	14.04	14.97
1500	100	25	15	15.03	15.97
1600	100	25	16	16.03	16.97
1700	100	25	17	17.03	17.97
1800	100	25	18	18.03	18.97
1900	100	25	19	19.03	19.97
2000	100	25	20	20.02	20.98

The value of N corresponding to n_c is

$$N = 100\frac{1 - \cos\alpha}{\cos\alpha}. \tag{10}$$

Equations (9) and (10) can be solved for values of tan α corresponding to given values of N. The values given in Table 1 were obtained in this way.

Recent Publications of
The Engineering Experiment Station

Bulletins

NO.

335. A Photoelastic Study of Stresses in Gear Tooth Fillets, by T. J. Dolan and E. L. Broghamer. 1942. *Forty-five cents.*
336. Moments in I-Beam Bridges, by N. M. Newmark and C. P. Siess. 1942. *One dollar.*
337. Tests of Riveted and Welded Joints in Low-Alloy Structural Steels, by W. M. Wilson, W. H. Bruckner, and T. H. McCrackin, Jr. 1942. *Eighty cents.*
338. Influence Charts for Computation of Stresses in Elastic Foundations, by N. M. Newmark. 1942. *Thirty-five cents.*
339. Properties and Applications of Phase-Shifted Rectified Sine Waves, by J. T. Tykociner and L. R. Bloom. 1942. *Sixty cents.*
340. Loss of Head in Flow of Fluids through Various Types of One-and-one-half-inch Valves, by W. M. Lansford. 1942. *Forty cents.*
341. The Effect of Cold Drawing on the Mechanical Properties of Welded Steel Tubing, by W. E. Black. 1942. *Forty cents.*
342. Pressure Losses in Registers and Stackheads in Forced Warm-Air Heating, by A. P. Kratz and S. Konzo. 1942. *Sixty-five cents.*
343. Tests of Composite Timber and Concrete Beams, by F. E. Richart and C. E. Williams, Jr. 1943. *Seventy cents.*
344. Fatigue Tests of Commercial Butt Welds in Structural Steel Plates, by W. M. Wilson, W. H. Bruckner, T. H. McCrackin, Jr., and H. C. Beede. 1943. *One dollar.*
345. Ultimate Strength of Reinforced Concrete Beams as Related to the Plasticity Ratio of Concrete, by V. P. Jensen. 1943. *Seventy cents.*
346. Highway Slab-Bridges with Curbs: Laboratory Tests and Proposed Design Method, by V. P. Jensen, R. W. Kluge, and C. B. Williams, Jr. 1943. *Ninety cents.*
347. Fracture and Ductility of Lead and Lead Alloys for Cable Sheathing, by H. F. Moore and C. W. Dollins. 1943. *Seventy cents.*
348. Fuel Savings Resulting from Closing of Rooms and from Use of a Fireplace, by S. Konzo and W. S. Harris. 1943. *Forty cents.*
349. Performance of a Hot-Water Heating System in the I=B=R Research Home at the University of Illinois, by A. P. Kratz, W. S. Harris, M. K. Fahnestock, and R. J. Martin. 1944. *Seventy-five cents.*
350. Fatigue Strength of Fillet-Weld and Plug-Weld Connections in Steel Structural Members, by W. M. Wilson, W. H. Bruckner, J. E. Duberg, and H. C. Beede. 1944. *One dollar.*
351. Temperature Drop in Ducts for Forced-Air Heating Systems, by A. P. Kratz, S. Konzo, and R. B. Engdahl. 1944. *Sixty-five cents.*
352. Impact on Railway Bridges, by C. T. G. Looney. 1944. *One dollar.*
353. An Analysis of the Motion of a Rigid Body, by E. W. Suppiger. 1944. *Seventy-five cents.*
354. The Viscosity of Gases at High Pressures, by E. W. Comings, B. J. Mayland, and R. S. Egly. 1944. *Free upon request.*
355. Fuel Savings Resulting from Use of Insulation and Storm Windows, by A. P. Kratz and S. Konzo. 1944. *Forty cents.*
356. Heat Emission and Friction Heads of Hot-Water Radiators and Convectors, by F. E. Giesecke and A. P. Kratz. 1945. *Fifty cents.*
357. The Bonding Action of Clays; Part I, Clays in Green Molding Sand, by R. E. Grim and F. L. Cuthbert. 1945. *Free upon request.*
358. A Study of Radiant Baseboard Heating in the I=B=R Research Home, by A. P. Kratz and W. S. Harris. 1945. *Thirty-five cents.*
359. Grain Sizes Produced by Recrystallization and Coalescence in Cold-Rolled Cartridge Brass, by H. L. Walker. 1945. *Free upon request.*
360. Investigation of the Strength of Riveted Joints in Copper Sheets, by W. M. Wilson and A. M. Ozelsel. 1945. *Free upon request.*

Bulletins (Continued)

NO.

361. Residual Stresses in Welded Structures, by W. M. Wilson and Chao-Chien Hao. 1946. *Seventy cents.*
362. The Bonding Action of Clay; Part II—Clays in Dry Molding Sand, by R. E. Grim and F. L. Cuthbert. 1946. *Free upon request.*
363. Studies of Slab and Beam Highway Bridges; Part I—Tests of Simple-Span Right I-Beam Bridges, by N. M. Newmark, C. P. Siess, and R. R. Penman. 1946. *Free upon request.*
364. Steam Turbine Blade Deposits, by F. G. Straub. 1946. *Free upon request.*
365. Experience in Illinois with Joints in Concrete Pavements, by J. S. Crandell, V. L. Glover, W. C. Huntington, J. D. Lindsay, F. E. Richart, and C. C. Wiley. 1946. *In press.*
366. Performance of an Indirect Storage Type of Hot-Water Heater, by A. P. Kratz and W. S. Harris. 1947. *Free upon request.*
367. Influence Charts for Computation of Vertical Displacements in Elastic Foundations, by N. M. Newmark. 1947. *Free upon request.*

Circulars

NO.

42. Papers Presented at the Twenty-eighth Annual Conference on Highway Engineering, held at the University of Illinois March 5-7, 1941. 1942. *Free upon request.*
43. Papers Presented at the Sixth Short Course in Coal Utilization, held at the University of Illinois May 21-23, 1941. 1942. *Free upon request.*
44. Combustion Efficiencies as Related to Performance of Domestic Heating Plants, by A. P. Kratz, S. Konzo, and D. W. Thomson. 1942. *Forty cents.*
45. Simplified Procedure for Selecting Capacities of Duct Systems for Gravity Warm-Air Heating Plants, by A. P. Kratz and S. Konzo. 1942. *Fifty-five cents.*
46. Hand-Firing of Bituminous Coal in the Home, by A. P. Kratz, J. R. Fellows, and J. C. Miles. 1942. *Free upon request.*
47. Save Fuel for Victory. 1942. *Free upon request.*
48. Magnetron Oscillator for Instruction and Research in Microwave Techniques, by J. T. Tykociner and L. R. Bloom. 1944. *Forty cents.*
49. The Drainage of Airports, by W. W. Horner. 1944. *Fifty cents.*
50. Bibliography of Electro-Organic Chemistry, by S. Swann, Jr. 1945. *In press.*
51. Rating Equations for Hand-Fired Warm-Air Furnaces, by A. P. Kratz, S. Konzo, and J. A. Henry. 1945. *Sixty cents.*

Reprints

NO.

26. First Progress Report of the Investigation of Fatigue Failures in Rail Joint Bars, by N. J. Alleman. 1943. *Free upon request.*
27. A Brief History of Lime, Cement, Concrete, and Reinforced Concrete, by J. O. Draffin. 1943. *Free upon request.*
28. Tenth Progress Report of the Joint Investigation of Fissures in Railroad Rails, by R. E. Cramer and R. S. Jensen. 1944. *Free upon request.*
29. Second Progress Report of the Investigation of Shelly Spots in Railroad Rails, by R. E. Cramer. 1944. *Free upon request.*
30. Second Progress Report of the Investigation of Fatigue Failures in Rail Joint Bars, by N. J. Alleman. 1944. *Free upon request.*
31. Principles of Heat Treating Steel, by H. L. Walker. 1944. *Fifteen cents.*
32. Progress Reports of Investigation of Railroad Rails and Joint Bars, by H. F. Moore, R. E. Cramer, N. J. Alleman, and R. S. Jensen. 1945. *Free upon request.*
33. Progress Report on the Effect of the Ratio of Wheel Diameter to Wheel Load on Extent of Rail Damage, by N. J. Alleman. 1945. *Free upon request.*
34. Progress Report of the Joint Investigation of Methods of Roadbed Stabilization, by R. B. Peck. 1946. *Free upon request.*
35. Progress Reports of Investigation of Railroad Rails and Joint Bars, by R. E. Cramer, N. J. Alleman, and R. S. Jensen. 1946. *Free upon request.*
36. Electro-Organic Chemical Preparations; Part III, by S. Swann, Jr. 1947. *Free upon request.*

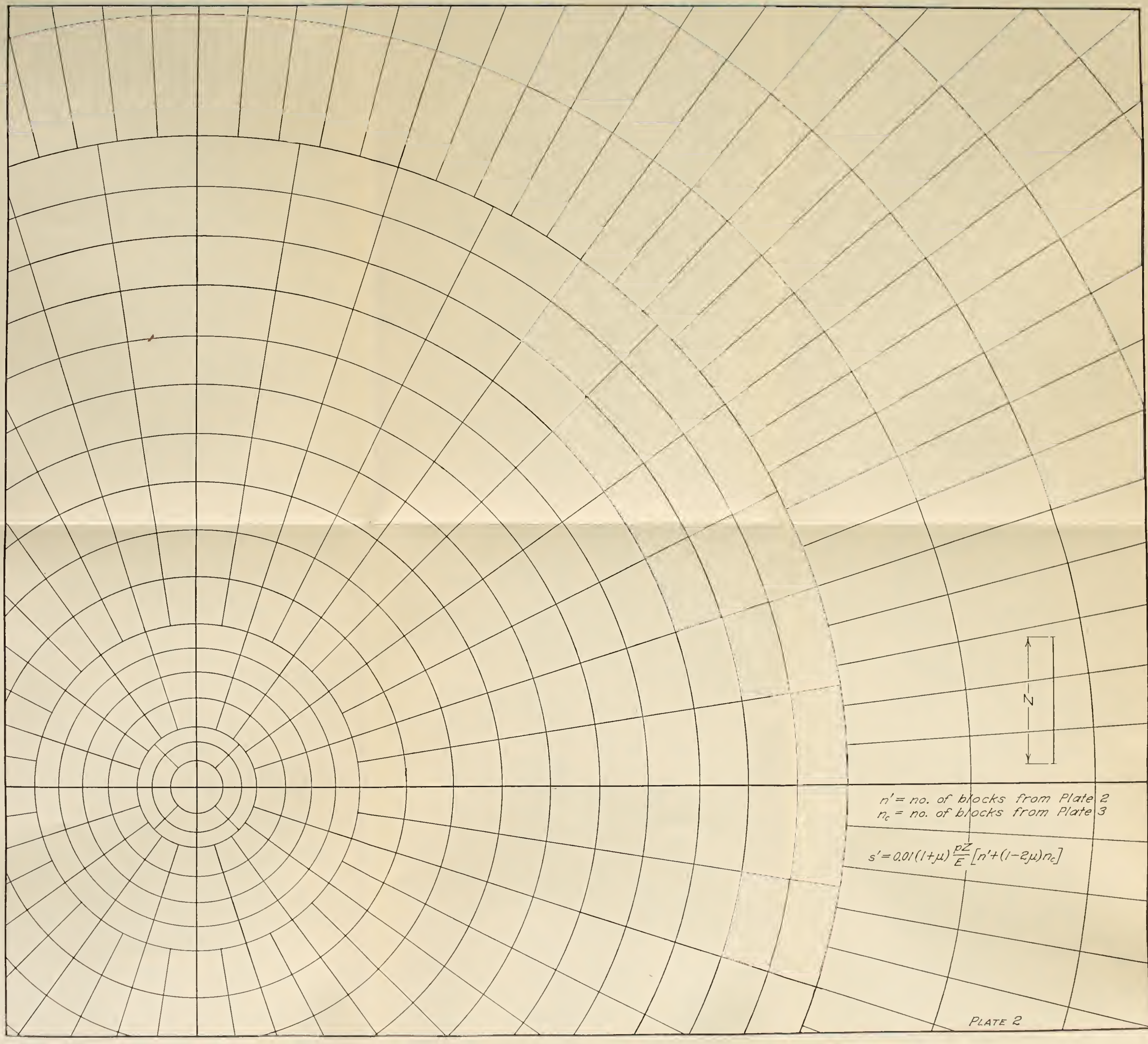

PLATE 2. INFLUENCE CHART FOR VERTICAL DISPLACEMENT AT DEPTH Z BENEATH SURFACE, POISSON'S RATIO = 0.5

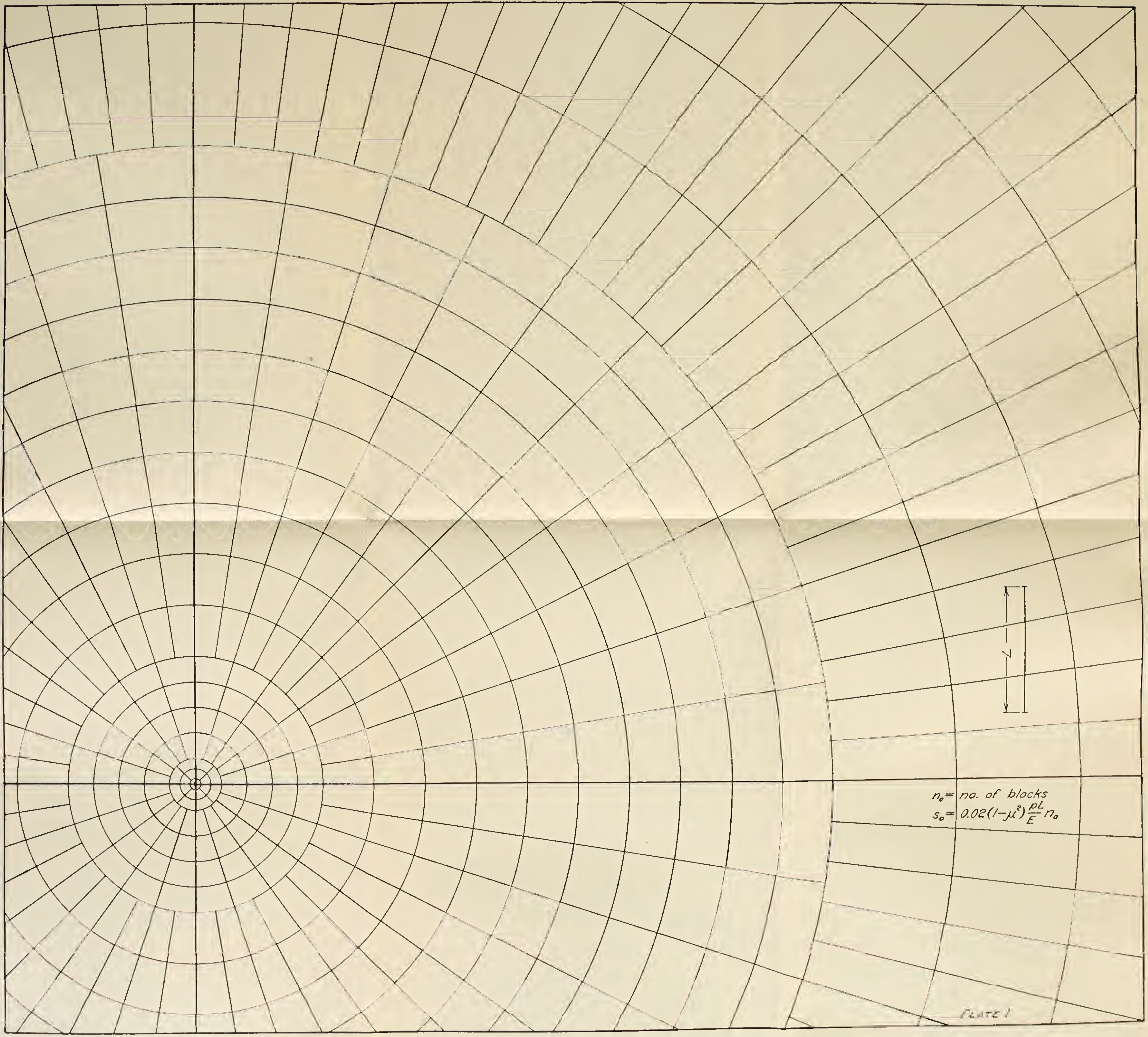

Plate 1. Influence Chart for Vertical Displacement at Surface

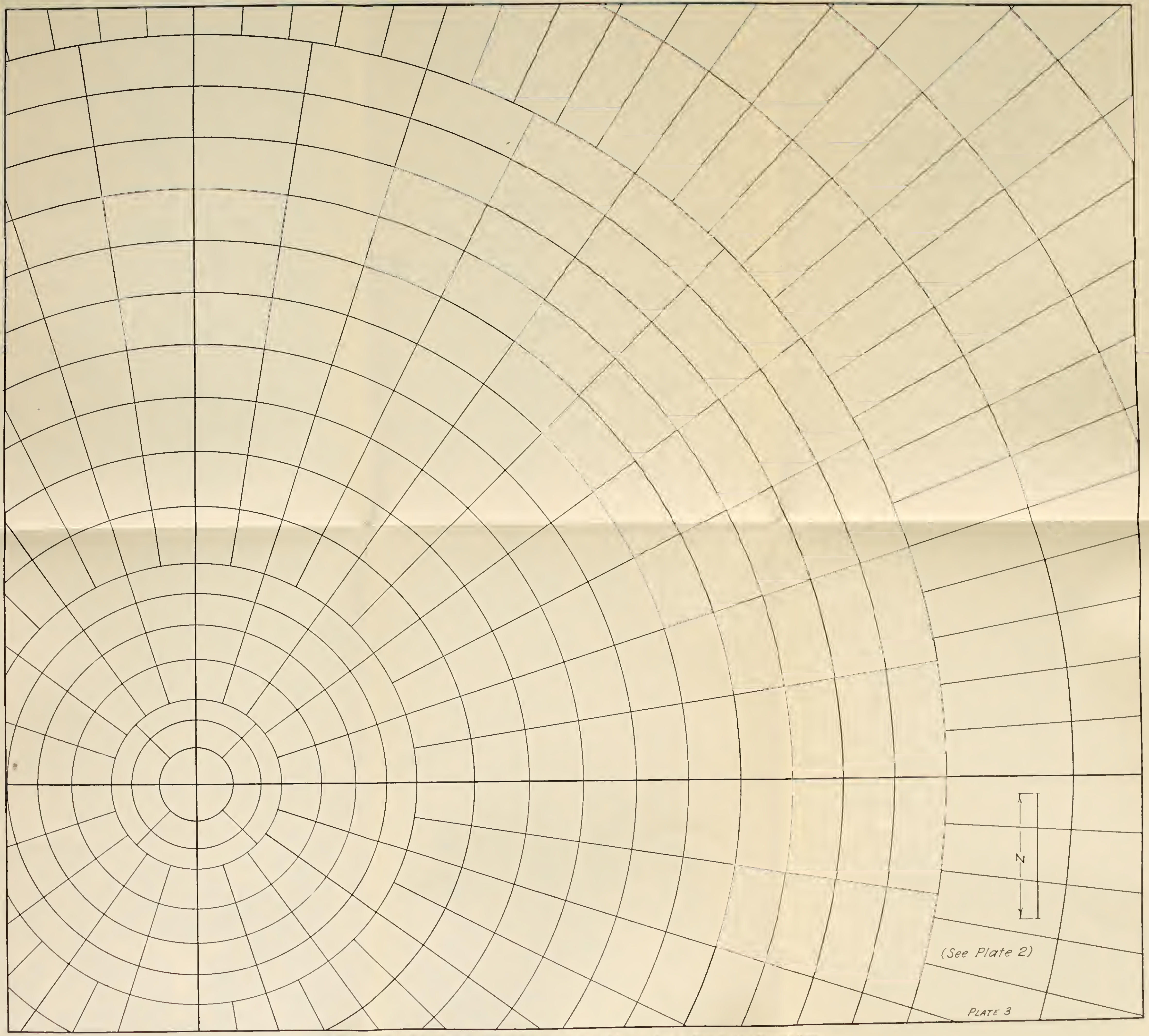

Plate 3 Influence Chart for Vertical Displacement at Depth Z Beneath Surface, Correction for Poisson's Ratio

UNIVERSITY OF ILLINOIS

Divisions of Instruction

- Institute of Aeronautics
- College of Agriculture
- College of Commerce and Business Administration
- College of Dentistry
- College of Education
- College of Engineering
- College of Fine and Applied Arts
- Graduate School
- School of Journalism
- Institute of Labor and Industrial Relations
- College of Law
- College of Liberal Arts and Sciences
- Library School
- College of Medicine
- Department of Military Science and Tactics
- Department of Naval Science
- College of Pharmacy
- School of Physical Education
- Division of Social Welfare Administration
- Division of Special Services for War Veterans
- Summer Term
- University Extension Division
- College of Veterinary Medicine

University Experiment Stations and Research and Service Organizations at Urbana

- Agricultural Experiment Station
- Bureau of Community Planning
- Bureau of Economic and Business Research
- Bureau of Educational Research
- Bureau of Institutional Research
- Engineering Experiment Station
- Extension Service in Agriculture and Home Economics
- General Placement Bureau
- High School Testing Bureau
- Radio Station (WILL)
- Services for Crippled Children
- Small Homes Council
- Student Personnel Bureau
- University of Illinois Press

State Scientific Surveys and Other Divisions at Urbana

- State Geological Survey
- State Natural History Survey
- State Water Survey
- State Diagnostic Laboratory (for Animal Pathology)
- U. S. Regional Soybean Laboratory

For general catalog of the University, special circulars, and other information, address The Registrar, University of Illinois, Urbana, Illinois